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2668. Proposed by B. F. FINKEL, Drury College.

Show that

$$v = \frac{2}{9}\frac{ga^2\sigma}{\eta},$$

where a is the radius of a droplet, σ its density, η the viscosity of the air and v the velocity under gravity g. Stokes's Law.

2669. Proposed by S. A. COREY, Albia, Iowa.

Let A_1, A_2, $\cdots$, A_8, and $-(A_1 + A_2 + \cdots + A_8)$ be the vector sides of an enneagon, plane or gauche. Also let B_1, B_2, $\cdots$, B_8, and $-(B_1 + B_2 + \cdots + B_8)$ be the vector sides of a second enneagon, where

$$\begin{aligned}
B_1 &= C_1A_1 - C_2C_5A_3 - C_3C_6A_5 + C_4C_5C_6A_7,\\
B_2 &= C_1A_2 - C_2C_5A_4 - C_3C_6A_6 + C_4C_5C_6A_8,\\
B_3 &= C_2A_1 + C_1A_3 - C_4C_6A_5 - C_3C_6A_7,\\
B_4 &= C_2A_2 + C_1A_4 - C_4C_6A_6 - C_3C_6A_8,\\
B_5 &= C_3A_1 + C_4C_5A_3 + C_1A_5 + C_2C_5A_7,\\
B_6 &= C_3A_2 + C_4C_5A_4 + C_1A_6 + C_2C_5A_8,\\
B_7 &= C_4A_1 - C_3A_3 + C_2A_5 - C_1A_7,\\
B_8 &= C_4A_2 - C_3A_4 + C_2A_6 - C_1A_8,
\end{aligned}$$

C_1, C_2, C_3, C_4, C_5, and C_6 being scalars.

Then, if a_s = tensor A_s, b_s = tensor B_s, and $\cos(A_rA_s)$ = cosine of the angle included between A_r and A_s, and $\cos(B_rB_s)$ = cosine of the angle included between B_r and B_s, establish the following relation between the sides and angles of the two enneagons:

$$\begin{aligned}
&[C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2]\,[a_1a_2 \cos(A_1A_2) + C_5a_3a_4 \cos(A_3A_4)\\
&\qquad + C_6a_5a_6 \cos(A_5A_6) + C_5C_6a_7a_8 \cos(A_7A_8)]\\
&\qquad = b_1b_2 \cos(B_1B_2) + C_5b_3b_4 \cos(B_3B_4) + C_6b_5b_6 \cos(B_5B_6) + C_5C_6b_7b_8 \cos(B_7B_8).
\end{aligned}$$

Show that Geometry problem 506 is a special case of the foregoing. Give illustrative example, using triangle or other simple geometric figure, by assuming that some of the sides of the first enneagon are zero.

SOLUTIONS OF PROBLEMS.

482 (Algebra). Proposed by C. F. GUMMER, Kingston, Ont.

Find a necessary and sufficient condition that the infinite sequences of positive quantities $(a_1, a_2, \cdots)$ and $(b_1, b_2, \cdots)$ may be such that the series $a_1x_1 + a_2x_2 + \cdots$ and $b_1x_1 + b_2x_2 + \cdots$ either both converge or both diverge, when the x's are any positive quantities.

SOLUTION BY THE PROPOSER.

The condition is that a_1/b_1, a_2/b_2, $\cdots$ all lie between two positive limits m and M $(m < M)$. It is sufficient; for it makes $m < (a_rx_r)/(b_rx_r) < M$. It is necessary; for if it does not hold, either the sequence $(a_1/b_1, a_2/b_2, \cdots)$ or $(b_1/a_1, b_2/a_2, \cdots)$ has $+\infty$ for one of its limits. That is (taking the first case), there is a partial sequence $(a_{i_1}/b_{i_1}, a_{i_2}/b_{i_2}, \cdots)$ of increasing quantities tending to ∞. Hence, using an argument due to DuBois-Reymond, the series

$$\sum_{r=2}^{\infty}\left(\sqrt{\frac{b_{i_{r-1}}}{a_{i_{r-1}}}} - \sqrt{\frac{b_{i_r}}{a_{i_r}}}\right)$$

converges, and the series

$$\sum_{r=2}^{\infty}\frac{a_{i_r}}{b_{i_r}}\left(\sqrt{\frac{b_{i_{r-1}}}{a_{i_{r-1}}}} - \sqrt{\frac{b_{i_r}}{a_{i_r}}}\right)$$

diverges. Hence, if

$$x_{i_r} = \frac{1}{b_{i_r}}\left(\sqrt{\frac{b_{i_{r-1}}}{a_{i_{r-1}}}} - \sqrt{\frac{b_{i_r}}{a_{i_r}}}\right) \qquad (r = 2, 3, \cdots),$$

the partial series $\Sigma_r a_{i_r} x_{i_r}$ diverges while $\Sigma_r b_{i_r} x_{i_r}$ converges. Also the other x's may be chosen so small as to make the corresponding values of bx less than the terms of a given convergent series; so that we have made one series diverge while the other converges.

Also solved by S. BEATTY.

483 (Algebra). Proposed by C. R. DUNCAN, Amherst College.

Prove or disprove the following theorem: An infinite series $A_1 + A_2 + A_3 + \cdots + A_n + \cdots$ is convergent or divergent according as

$$\lim_{n=\infty} \frac{A_n}{1 - \dfrac{A_n}{A_{n-1}}} = 0 \qquad \text{or} \qquad \neq 0.$$

SOLUTION BY EDWIN BIDWELL WILSON, Massachusetts Institute of Technology.

The test when applied to the series

$$1 + \tfrac{1}{1} + 2 + \tfrac{1}{2} + 3 + \tfrac{1}{3} + \cdots + n + \frac{1}{n} + \cdots$$

indicates convergence, whereas the series clearly diverges. If this example be considered artificial consider the series

$$\frac{1}{2(\log 2)^p} + \frac{1}{3(\log 3)^p} + \cdots + \frac{1}{n(\log n)^p} + \cdots.$$

Cauchy's integral test shows that this series diverges when $p \leqq 1$ and converges when $p > 1$.

$$\frac{A_n}{1 - \dfrac{A_n}{A_{n-1}}} = \frac{1}{\dfrac{1}{A_n} - \dfrac{1}{A_{n-1}}} = \frac{1}{n(\log n)^p - (n-1)[\log (n-1)]^p}.$$

Now

$$\frac{1}{(\log n)^p} < \frac{1}{n(\log n)^p - (n-1)[\log (n-1)]^p} < \frac{1}{[\log (n-1)]^p}.$$

Hence, the limit sought by the test is zero for all positive values of p, whereas it should be zero only for values greater than unity.

The suggested test for convergence is therefore no good.

If the limit exists and is greater than zero, then

$$\frac{1}{\dfrac{1}{A_n} - \dfrac{1}{A_{n-1}}} > \gamma \qquad \text{or} \qquad \frac{1}{A_n} - \frac{1}{A_{n-1}} < \frac{1}{\gamma}.$$

Hence

$$\frac{1}{A_{n+p}} - \frac{1}{A_{n-1}} < \frac{p+1}{\gamma},$$

$$A_{n+p} > \frac{1}{\dfrac{1}{A_{n-1}} + \dfrac{p+1}{\gamma}} > \frac{\gamma}{\dfrac{p+1+\gamma}{A_{n-1}}}.$$

(We have assumed a series of positive terms.) The series, therefore, diverges by comparison with the harmonic series. Hence, the test for divergence when applied to series of positive terms is valid.

Also solved by HORACE OLSON.

514 (Geometry). Proposed by VICENTE MILLS, Manila, P. I.

Given an equilateral triangle, the length of the sides being unknown, and a point within, the distances from which to the vertices are given, required the length of a side of the triangle and the angles subtended at the given point by the sides of the triangle.